Math 60

9.8 Radical Equations and their Applications (2 days)

Objectives day 1 { 1) Solve radical equations containing one radical.

day 2 { 2) Solve radical equations containing two radicals. (next class)

3) Solve for a variable in a radical equation.

4) Solve equations containing a rational exponent.

Warm up:

Simplify.

① $(\sqrt{2x-3})^2$

$= \boxed{2x-3}$

Note: $\sqrt{(2x-3)^2} = |2x-3|$

But this does not concern us in 9.8

② $(\sqrt{2x} + \sqrt{3})^2$

$\uparrow$

+ symbol means 2 terms, FOIL

$= (\sqrt{2x} + \sqrt{3})(\sqrt{2x} + \sqrt{3})$

$= 2x + \sqrt{6x} + \sqrt{6x} + 3$

$= \boxed{2x + 3 + 2\sqrt{6x}}$

Notice: If we square one term, the radical is removed.
But if we square two terms, the radical is not removed.

③ Solve $\dfrac{2}{a-1} + \dfrac{3}{a+1} = \dfrac{-6}{a^2-1}$

Remember: $a \neq 1$ and $a \neq -1$

If we get $a=1$ or $a=-1$ we reject them as extraneous solutions.

③ $\dfrac{2}{a-1} + \dfrac{3}{a+1} = \dfrac{-6}{(a+1)(a-1)}$

mult all by LCD $= (a+1)(a-1)$

$$\frac{2(a+1)\cancel{(a-1)}}{\cancel{(a-1)}} + \frac{3\cancel{(a+1)}(a-1)}{\cancel{(a+1)}} = \frac{-6\cancel{(a+1)}\cancel{(a-1)}}{\cancel{(a+1)}\cancel{(a-1)}}$$

$2(a+1) + 3(a-1) = -6$

$2a+2+3a-3 = -6$

$5a-1 = -6$

$5a = -5$

$a = 1$ reject

$\boxed{\phi}$

Solving a rational equation is not related to today's lesson – but radical equations can also have extraneous solutions.

④ Solve $\sqrt{2x-3} - 5 = 0$

Step 1: Isolate the radical. (add 5 both sides)

$\sqrt{2x-3} = 5$

Step 2: Square entire LHS, entire RHS, using parentheses if two or more terms/signs

$(\sqrt{2x-3})^2 = (5)^2$

$2x-3 = 25$

Step 3 : Isolate the variable

$$\frac{2x}{2} = \frac{28}{2}$$

$$x = 14$$

Step 4: Not optional! Plug in to check if answer is extraneous. Must use original equation, before we squared both sides.

$$\sqrt{2x-3} - 5 \stackrel{?}{=} 0$$

$$\sqrt{2(14)-3} - 5 = 0$$

$$\sqrt{28-3} - 5 = 0$$

$$\sqrt{25} - 5 = 0$$

$$5 - 5 = 0 \checkmark$$

$\boxed{x = 14}$ is a valid solution.

5 Solve $\sqrt{5x+6} - 3 = -2$

isolate radical

$$\sqrt{5x+6} = 1$$

$$(\sqrt{5x+6})^2 = 1^2$$

$$5x + 6 = 1$$

$$5x = -5$$

$$x = -1$$

check $\sqrt{5(-1)+6} - 3 = -2$

$$\sqrt{-5+6} - 3 = -2$$

$$\sqrt{1} - 3 = -2$$

$$1 - 3 = -2 \checkmark$$

$$\boxed{x = -1}$$

Math 60 9.8

(6) Solve $\sqrt{3x-5}+8=3$

$$\sqrt{3x-5}=-5$$

isolate radical

$$\left(\sqrt{3x-5}\right)^2=(-5)^2$$

Wide awake? Remember that $\sqrt{\ }$ means the principle square root, which is positive.
$\Rightarrow$ $\boxed{\text{no solution}}$

Sound asleep?
Square both sides
Notice $(-5)^2$ was negative, but becomes positive.
This is how extraneous solutions are created.

$$3x-5=25$$
$$3x=30$$
$$x=10$$

check $\sqrt{3(10)-5}+8\overset{?}{=}3$

$$\sqrt{30-5}+8=3$$
$$\sqrt{25}+8=3$$
$$5+8\neq 3$$
$$13\neq 3$$

reject! $\boxed{\text{no solution}}$

Math 60 9.8

(7) Solve $\sqrt{3x-11} = x-5$

step 1: radical is already isolated.

step 2: square both sides, using parentheses

$$(\sqrt{3x-11})^2 = (x-5)^2$$

IMPORTANT

$$(x-5)^2 \neq x^2-5^2$$

$$(x-5)^2 = (x-5)(x-5)$$

$$= x^2-10x+25$$

Step 3: FOIL to simplify RHS

$$3x-11 = x^2-10x+25$$

step 4: Notice the type of equation. This is x^2, quadratic.

To solve a quadratic

1) set $=0$.
2) factor completely
3) set factors $=0$.

$$0 = x^2-10x+25-3x+11$$

$$x^2-13x+36 = 0$$

$$(x-9)(x-4) = 0$$

36, −9, −4, −13

$$x = 9, 4$$

step 5 Check both answers in original equation.

$x=9$: $\sqrt{3(9)-11} = 9-5$

substitute $x=9$ in both locations

$$\sqrt{27-11} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4$$

Keep $x=9$.

$x=4 \quad \sqrt{3(4)-11} = 4-5$

$\sqrt{12-11} = -1$

$\sqrt{1} = -1$

$1 \neq -1$ reject $x=4$.

$\boxed{x=9}$ is the only valid solution.

8. Solve $\sqrt[3]{p^2-4p-4} = \sqrt[3]{-3p+2}$

Notice: cube roots, index 3.
This means two important things:
1) We will raise both sides to the 3rd power ("cube both sides") instead of 2nd power.
2) $\sqrt[3]{neg}$ is a valid result, so we will not have to check for extraneous results.

step 1: cube both sides

$$\left(\sqrt[3]{p^2-4p-4}\right)^3 = \left(\sqrt[3]{-3p+2}\right)^3$$

$$p^2-4p-4 = -3p+2$$

step 2: Notice $p^2 \Rightarrow$ quadratic. Set $=0$, factor

$$p^2-p-6=0$$

-6, -3, $+2$, -1

$$(p-3)(p+2)=0$$

$$\boxed{p=3 \quad p=-2}$$

In general: If the index is even, we must check for extraneous solutions. If the index is odd, checking is optional and only to check our work.